On the Zeta Functions of Supersingular curves

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Abstract

The zeta function of a curve C defined over a finite field $\mathbb{F}_q$ is the formal power series

$$\exp\left(\sum_{n=1}^{\infty} \#C(\mathbb{F}_{q^n})\frac{T^n}{n}\right).$$

It is known that the zeta function is a rational function, with only the numerator depending on C. The numerator is called the L-polynomial of C, and is therefore equivalent to knowledge of the number of points $\#C(\mathbb{F}_{q^n})$ on the curve over all extensions of $\mathbb{F}_q$.

In general, the L-polynomial of a curve of genus g is determined by g coefficients. We show that the L-polynomial of a supersingular curve of genus g is determined by fewer than g coefficients. We characterize which coefficients are needed.

Our result gives precise information about the number of rational points. We will give some applications of our result to Artin-Schreier curves and Hermitian curves.

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